Theme 8. Electrostatics and electrodynamics basics

Problem. Human structures functions identification and quantification as electric processes to get evidenced data of structures functional states

Attendance prerequisite checklist. Note! Answer in writing to perform

Answer to the questions
1. Define physics phenomenon “conservative force”.
2. Define and write down parameters of a parallel plate capacitor electric field.
3. Explain the meaning of Physics term “electric current”.
4. Why human cell membrane may be considered as a capacitor?

Information resources
1. http://www.a-levelphysicstutor.com/elect-ebook.php
2. https://www.slideserve.com/jaser/the-electric-dipole

Electrostatics

Our study of electricity begins with electrostatics and the electrostatic force, one of the four fundamental forces of nature. Electrostatic force is described by Coulomb's Law. We use Coulomb's Law to solve the forces created by configurations of charge.

Electrostatics deals with forces between **charges.** “Static” means the charges are not moving, or at least are not moving very fast.

Charge. How do we know there is such a thing as charge? The concept of charge arises from an observation of nature: we observe forces between objects. Electric charge is the property of objects that gives rise to this observed force. Like gravity, electric force "acts at a distance". The idea that a force can "act at a distance" is pretty mind-blowing, but it's what nature really does.

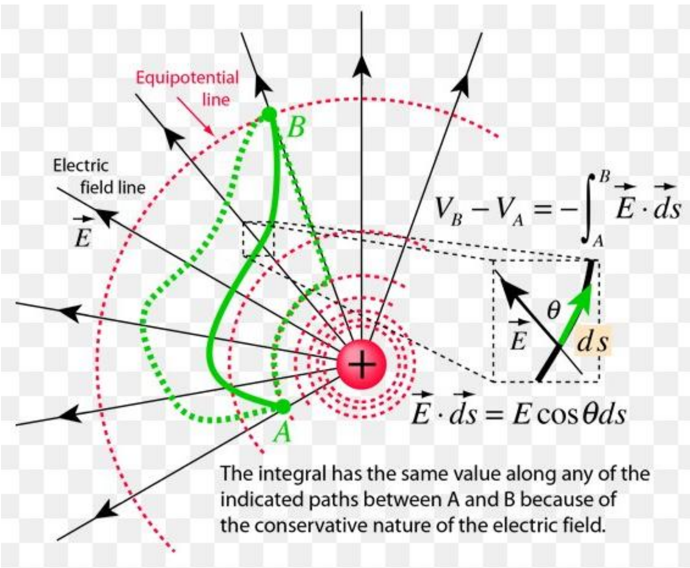

Figure 1

Electric forces are very large, far greater than the force of gravity. Unlike gravity, there are two types of electric charge, whereas there is only one type of gravity; gravity only attracts. Unlike charges attract, and like charges repel.

On the sub-atomic level we have a unit of charge of 'e', the charge on the electron. This has the value $-1.602\cdot10^{-19}$C. Similarly, a proton has a charge of $+1.602\cdot10^{-19}$C.

The macro unit of charge in electricity is the **Coulomb** (C). It is quite a large unit and has approximately the same charge as $6.2\cdot10^{18}$ electrons, or protons.

Electric charges produce **electric field** in their vicinity. If another charge **q** is present, it experiences a force proportional to the electric field **E** and to **q** itself: **F = qE**. Since the force **F** is a vector, the electric field **E** is also a vector. Positive charges experience forces parallel to the field, and negative charges experience forces opposite to the field.

When there are several charges at various positions, electric field at any point is the vector sum of the individual electric fields due to all to charges. Once we have measured the electric field at a point, we can immediately find the force on any charge placed there. It is not necessary to know the magnitude or location of the charges producing the field. Any force that has the property that the work it does is the same for all paths between any two given points is said to be a **conservative force**. The electrical force is conservative. This property makes it meaningful to associate a **potential energy** with a position. The potential energy of two charges q and Q separate by distance r is $U = kqQ/r$.

The electrical potential is the potential energy per unit charge: $V = U/q$. The unit of potential is volt (V).

The potential difference (V_B - V_A) is referred to as **voltage (**fig.1).

Potential difference is short for **potential energy** difference. Our definition of the volt relates charge and the work needed to move that charge between two points. In the definition of the 1 volt, the charge is 1 Coulomb and the work done is 1 Joule.

Two points A and B are at a potential difference of 1 volt if the work required to move 1 coulomb of charge between them is 1 Joule (fig. 2).

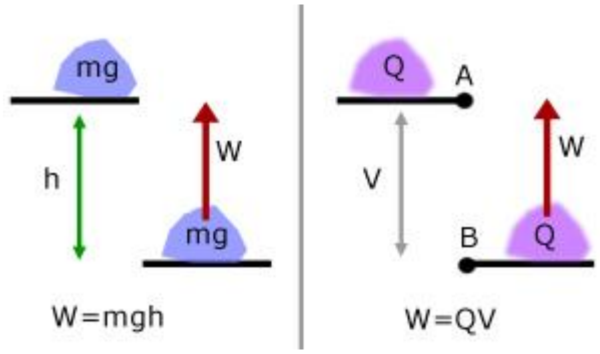

Figure 2

A simple analogy is gravity. Compare the energy difference between a rock at the bottom of a cliff and the energy in moving it to the top of the cliff. An energy difference exists between the top of the cliff and the bottom. The amount of energy difference depends on the size of the rock and the height of the cliff.

In our analogy, height relates to potential difference and rock weight relates to the amount of charge.

Work is done on the rock against the force of gravity. Work is done on the charge in moving it against an electrostatic force.

We now have an equation linking work/energy **W**, charge **Q** and potential difference **V**: $V = W/Q$; $W = V \cdot Q$. The volt has unit of joules per coulomb (JC^{-1}).

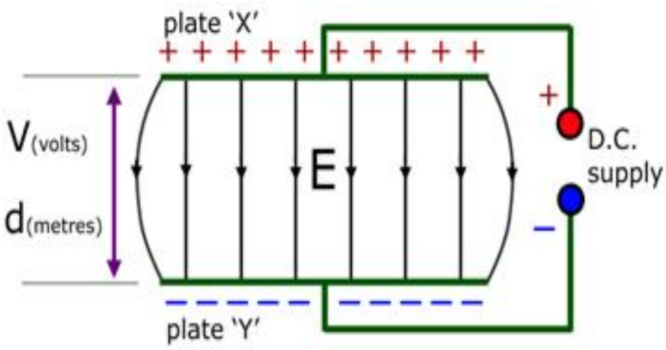

Figure 3

Uniform Electric Field. Two oppositely charged and parallel metal plates X and Y with constant potential difference along their length ($V_X - V_Y$) (fig. 3) produce a uniform electric field **E** between them: $\mathbf{E = (V_X - V_Y)/d}$. The field is directed from the positive plate at the higher potential toward the negative plate.

Note, at the edges the field lines are not evenly spaced. So the field there is not uniform.

An arrangement of two conductors separated by a vacuum or an insulator is called a **capacitor**. **The capacitance** is a measure of the amount charge separation that can be maintained at a given potential difference. The energy required to separate the charges is stored in the capacitor.

If two conductors (fig. 2) with constant potential difference along their length ($V_X - V_Y$) have equal and opposite charges ±Q, the ratio is the capacitance: $C = Q/(V_X - V_Y)$. Capacitance in case of a parallel plate capacitor (fig. 2) with equal plates surface areas A is: $C = \varepsilon_0 A/d$, $\varepsilon_0 = 8.85 \cdot 10^{-12}\ C^2/(N \cdot m^2)$. The unit of capacitance is the farad (F): 1 F = 1 C/1 V.

If an insulator or dielectric is introduced between the capacitor conductors with fixed charges, the capacitance is increased. For a parallel plate capacitor $C = \varepsilon\varepsilon_0 A/d$, $\varepsilon > 1$ is an insulator dielectric constant.

A charged capacitor stores **energy**: Energy = $\frac{1}{2}Q(V_X - V_Y) = \frac{1}{2}Q^2/C = \frac{1}{2}C(V_X - V_Y)^2$.

Electrodynamics

Electric current - symbol **I**, can be thought of as a flow of charged particles. In the normal case, when charge flows through wires these particles are electrons. However, when current flows in a vacuum, a solution or a melt, the charge carriers are ions. In semiconductors the charge carriers are exotic particles called 'holes'.

By definition the Ampere **A** is defined as the current passing an arbitrary fixed point when a charge of 1 Coulomb flows for 1 second.

Since, amount of fluid = rate of flow·time by analogy, charge = electric current·time, Coulombs = Amperes·seconds: $Q = I \cdot t$.

Resistance. By definition, the electrical resistance **R** of a conductor is the ratio of the potential difference V across it to the current I passing through it: $\mathbf{R = V/I}$.

The definition of the Ohm is: the resistance of a conductor through which a current of 1 ampere flows when a p.d. of 1 volt exists across it (fig. 4).

Note in the diagram, the voltmeter is in parallel with the resistance, while the ammeter is in series.

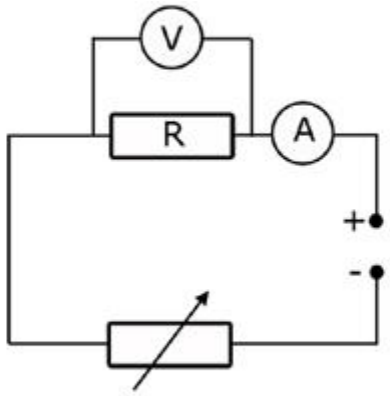

Figure 4

Ohm's Law. The current through a resistor is varied, while the potential difference across it is measured. The graph of V against I is a straight line through the origin. Hence, $V \propto I$, V/I = constant, gradient (R) = $(V_2 - V_1)/(I_2 - I_1)$ (fig 5).

Ohm's law states: The current through a conductor is directly proportional to the potential difference across it.

Resistivity. By experiment it has been found that the resistance R of a material is directly proportional to its length *l* and inversely proportional to its cross-sectional area A: $R \propto l/A$

Making the proportionality an equation, the constant of proportionality **ρ** is called the **resisitivity**: $R = \rho l/A$. Resistivity is measured in units of ohm·metres (Ω·.m).

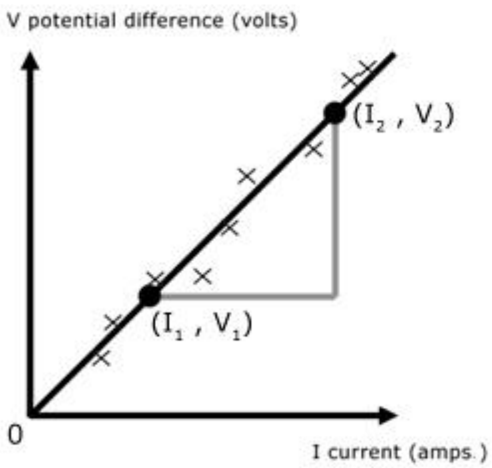

Figure 5

Conductance G. By definition, the conductance of a material is the reciprocal of its resistance: $G = 1/R$. The unit of conductance is the Seimens S.

Conductivity. By definition, the conductivity **σ** of a material is the reciprocal of its resistivity: $\sigma = 1/\rho$. The units of conductivity are Sm^{-1} or $\Omega^{-1}\cdot m^{-1}$.

Current density. Current density **J** at a point along a conductor, is defined as the current per unit cross-sectional area: $J = I/A$. The units of current density J are amps per m^2 or $A\cdot m^{-2}$.

Circuits containing resistance and capacitance. If we connect an uncharged capacitor to a battery, charge moves from one capacitor plate to the other through the battery and the connecting wires. This current stops once the potential difference across the capacitor equals the EMF of the battery. Similarly, if the plates of a charged capacitor are connected by a wire, there will be a current in the wire until the capacitor is fully discharged. These short-lived currents are referred as transients.

To study this more detail, we calculate the potential differences across the circuit elements during the charging process. Suppose at time t the charge on the capacitor is q and the current in the circuit is I (fig. 6). Proceeding counterclockwise, there is a potential rise, E, in the EMF, and a drop, iR, in the resistor. For the capacitor, C=q/V, so the potential drop is q/C. The sum of these changes is zero. Thus,

$E - iR - q/C = 0$, or $i = E/R - q/RC$

The charge and current at any time **t** can be found by solving last equation:

$$\mathbf{q = q_f \cdot (1 - e^{-t/T}),\ \ i = i_0\, e^{-t/T}}$$, where

$q_f = EC$, $i_0 = E/R$, $T = RC$, $e = 2.718$.

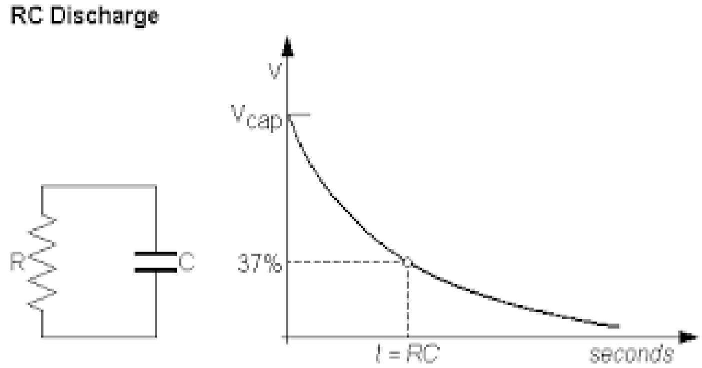

Figure 6

Note, that when t = T, $e^{-t/T} = e^{-1} = 1/2.718 = 0.37$. Thus, after one time constant **T** has elapsed, the current has dropped to about 37 percent of its initial value, i_0. Also, since $(1- e^{-1}) = 0.63$, after a time T has elapsed, $q = 0.63q_f$. That means the capacitor charge is equal to 63 percent of its final charge q_f.

The time required for either q or i to reach a specific fraction of its final value is proportional to time constant **T** given by **T = RC.**

A charged capacitor stores energy. If its plates are connected by conducting wire, electrons will move in the wire from the negative plate to the positive plate. This charge flow or current continues until the plates are neutralized and can be used to operate a trigger an artificial heart pacemaker. Thus, stored electrical energy can be transformed into other forms of energy.

The **energy stored** in a capacitor may initially be supplied by a battery, which maintains a potential difference between its two terminals. When the plates of an uncharged capacitor are connected by two wires to the terminals, electrons flow in the wires from one plate through the battery to the other until the potential difference across the capacitor reaches a maximum value. We can find the energy stored in the capacitor by calculating the work that must be done by the battery in building up this charge from zero to the final value q. If at some instantaneous the charges are $+q_i$ and $-q_i$, then from the definition of capacitance, $C= q_i/ U_i$, we have $U_i= q_i/C$. Transferring a small additional amount of charge Δq_i therefore requires that work ΔW be done, where $\Delta W = U_i\Delta q_i = q_i\Delta q_i/C$. Since the electric force is conservative, the work done by the battery must equal the increase ΔE in the stored energy of the capacitor, so $\Delta E_i = q_i\Delta q_i / C$. To find the total energy of the capacitor, we must sum the small increases in its energy ΔE_i associated with each charge transfer Δq_i. Thus the total **energy stored in the capacitor** is **E=1/2 QU.** Using C=Q/U, the energy can be expressed in three equivalent forms,

$$\mathbf{E = \tfrac{1}{2} QU = \tfrac{1}{2} Q^2/C = \tfrac{1}{2} CU^2}$$

Circuits containing resistance and inductance. Any circuit always has some inductance. The inductance retards current changes, but it has no effect on a current that is constant. We now examine this more detail for a series circuit consisting of a resistance R, an EMF E provided by a battery, and an inductance L.

When the switch is closed in the circuit, the inductance prevents a sudden change in the current. Instead, the current gradually increases from zero toward the current predicted by Ohm's law, i_f=E/R. The time constant T_L, which characterizes this buildup, is proportional to L, since the greater the inductance the longer it takes to achieve the final current. If the resistance R is large, then the magnitude of the overall current change in small, and the final current may be achieved more rapidly. Thus, we also expect T_L to be proportional to the 1/R, and the time constant is $T_L = L/R$.

This time constant plays a role in the RL series circuit that closely parallels that of the time constant T_C = RC in an RC series circuit. Suppose that the switch is closed at t=0. When $t=T_L$, the current differs from its final value i_f by $i_f e^{-1}=0.37 i_f$. At any time t, the current i is given by $i = i_f(1-e^{-t/T})$.

Just as a capacitor can store electrical energy, so also **an inductor can store magnetic energy**. To calculate this energy, we note that the power that must be supplied to change a current i in an inductor at a rate Δi/Δt is P=EMF x i = L(Δi/Δt)i. In a time Δt, the work done against the induced EMF is

$$\Delta W = P\Delta t = L(\Delta i/\Delta t)i\,\Delta t = Li\Delta i$$

Now the energy stored in an inductor with current i equals the work done in increasing the current from 0 to i. The **energy stored in an inductor** is $\mathbf{E = \frac{1}{2} Li^2}$.

Circuit consisting of a coil of inductance L and capacitor of capacitance C. Circuit consisting of a coil of inductance L and capacitor of capacitance C are called LC circuit. In these circuits electromagnetic oscillations can be produced if, for instance, an alternating current is applied. For this reason such circuit are frequently called oscillator circuits. An LC circuit will be resonant to the exciting voltage if its frequency is equal to the resonance frequency $\mathbf{f_0=1/(2\pi\sqrt{LC})}$ of the circuit. If either L or C both is varied, the resonance frequency of the circuit can be changed, that is, the LC circuit can be turned.

With a combination of elements with frequency-dependent impedance (L and C) and resistance R it is possible to construct filter circuits with any transfer characteristics: for instance, filter circuits transmitting in a determined frequency range.

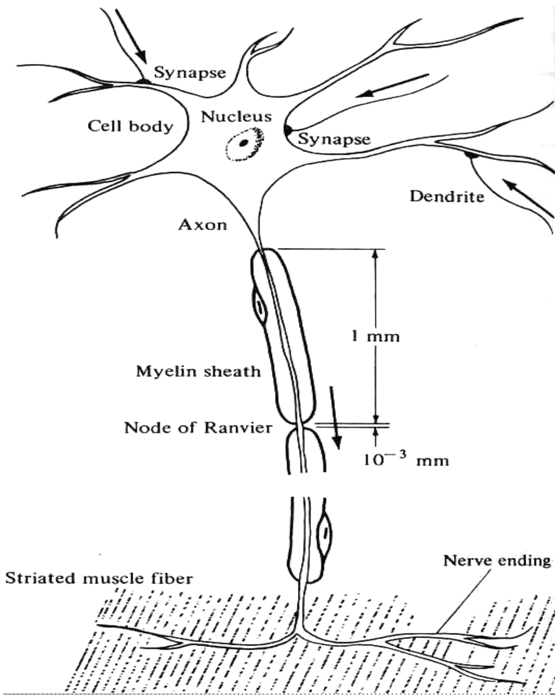

Figure 7

The resistance and capacitance of an axon. We can understand many of the electrical properties of an axon with the aid of a model that resembles an electrical cable covered with defective insulation so that current leaks to the surroundings in many places. More specifically, we assume the axon consists of cylindrical membrane containing a conducting fluid, the axoplasm. The current can travel along the axon in this fluid and can also leak out through the membrane.

The electrical properties of the axon are determined by several quantities. The **resistance R of a length of the axon to current $\mathbf{i_{axon}}$** along the axon is proportional to **axoplasm resistivity**, ρ_a. The **resistance of a unit area of membrane to a leakage current $\mathbf{i_{leak}}$** is labeled $\mathbf{R_m}$. The membrane also has **capacitance**, since charges of opposite signs accumulate on the two sides of the membrane. **The charge per unit area divided by resulting potential difference is the capacitance of a unit area, $\mathbf{C_m}$.**

From the figure 8a the resistance of a wire of length ***l***, cross-sectional area $\mathbf{A = \pi r^2}$, and resistivity $\boldsymbol{\rho_a}$ is $\mathbf{R = \rho_a l/A}$.

The membrane is quite thin, so a small section appears nearly flat. This permits us to use the **parallel plate capacitance formula**, which states that the capacitance is proportional to the plate area, **A**. A length ***l*** of membrane has a **surface area A = 2πrl**. Since the **capacitance per unit area** is C_m, the **capacitance of the length l** of axon is $C = C_m(2\pi rl)$ (fig. 8c)

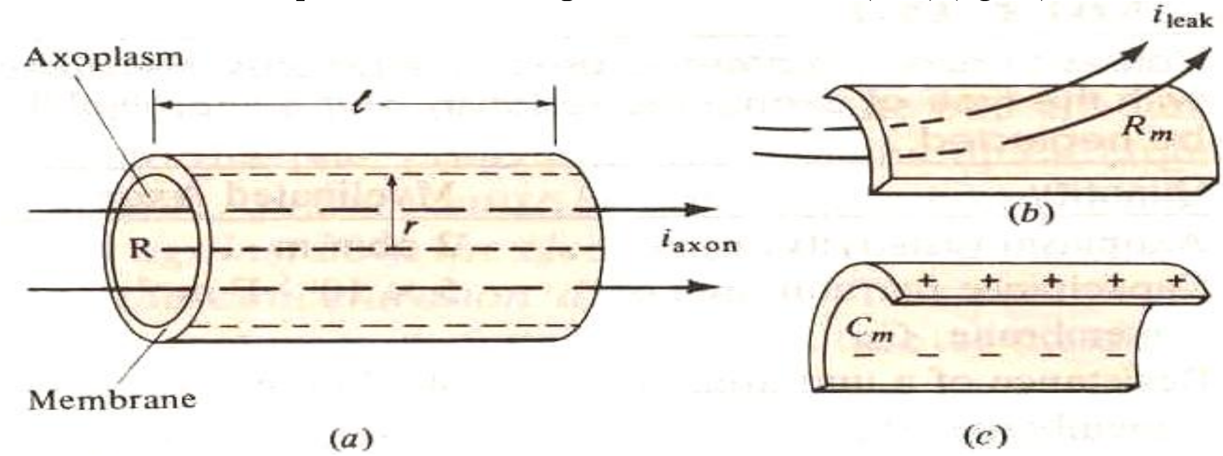

Figure 8

Since a membrane is not a perfect insulator, charge leaks from the axoplasm through the membrane into the interstitial fluid. The resistance of a conductor is inversely proportional to its cross-sectional area. If the **resistance to leakage currents through a unit surface area of membrane is R_m**, then a portion of the membrane with surface area **A** has a **resistance**
$R' = R_m/(2\pi rl)$ (fig.8b).

According to our model, the axoplasm resistance **R** is proportional to the length **l** of the axon segment, and the leakage resistance **R'** is proportional to **1/l**. Thus, there is some distance **λ** for which the resistances **R** and **R'** are equal. Using equations for **R** and **R'**, respectively, **λ** must satisfy:

$$\rho_a\lambda/(\pi r^2) = R_m/2\pi r\lambda, \text{ or } \lambda = (R_m r/2\rho_a)^{1/2}$$

The distance **λ**, called the **space parameter**, indicates how far current travels before most of it has leaked out through the membrane.

The response to weak stimuli. Having considered the resting state of the axon, we now examine the response of an axon to a weak stimulus. In most experiments, the stimuli are electrical, since these are easily controlled and do not injure the cell if they are sufficiently mild. For an electrical stimulus smaller than a critical threshold value, the response of the axon is similar to that of an analog network of resistors and capacitors. Specifically, if a weak stimulus is applied at some point on an axon, no significant axon potential changes occur beyond a few millimeters from that point. By contrast, a stimulus above the threshold level produces a current pulse that travels the length of the axon without attenuation. This current pulse and associated action potential are discussed later.

We can develop the analog circuit for the axon by dividing the axon into many short segments. The interstitial fluid surrounding the axon has very little resistance and may be represented by a perfect conductor. Each axon segment has a resistance **R** to a current i_{axon} along its length. The membrane has a resistance **R'** to a leakage current i_{leak} plus a capacitance **C** (fig. 9a, 9b). A series of several segments is then analogous to the complex network of resistors and capacitors in the figure 9c. The EMF shown there represents an applied stimulus.

The behavior of this complex axon analog circuit is more understood if we first consider the simplest **RC network** in the figure 9a. Suppose that the initial charge **q** on **C** is zero, and the switch is closed at **t = 0**. The charge **q** and the potential difference **v=q/C** will gradually increase. The time needed to reach a characteristic fraction of the final values of **q** and **v** is determined by the time constant, **T = RC.**

When there are two resistors and capacitors, as in Figure 21.5b, the charging process is more complicated. The potential difference v_2 across C_2 must increase more slowly than the potential difference v_1 across C_1, since the path from the battery to C_2 and back has resistance

2R. As more and more **RC pairs** are added, the potential difference across each added capacitor rises ever slowly. Consequently, in the axon analog circuit (fig. 9c), $\mathbf{v_2}$ will change more slowly than $\mathbf{v_1}$, $\mathbf{v_3}$ still more slowly, and so on.

The effect of a leakage resistor can be seen in the figure 10c. There is always some current in the conducting path through **R** and **R'**. Thus, there is a corresponding potential drop across **R**, and the final potential difference across the capacitor is less than the EMF. In the axon analog circuit, the final potential differences steadily diminish as we move to the right because of the current lost through the leakage resistors **R'**.

To summarize, when the switch is closed or a "stimulus" is applied in the axon analog circuit, the potential differences across the capacitors gradually change. As one goes farther from stimulus, the changes occur more slowly, and their final magnitudes diminish (figure 11).

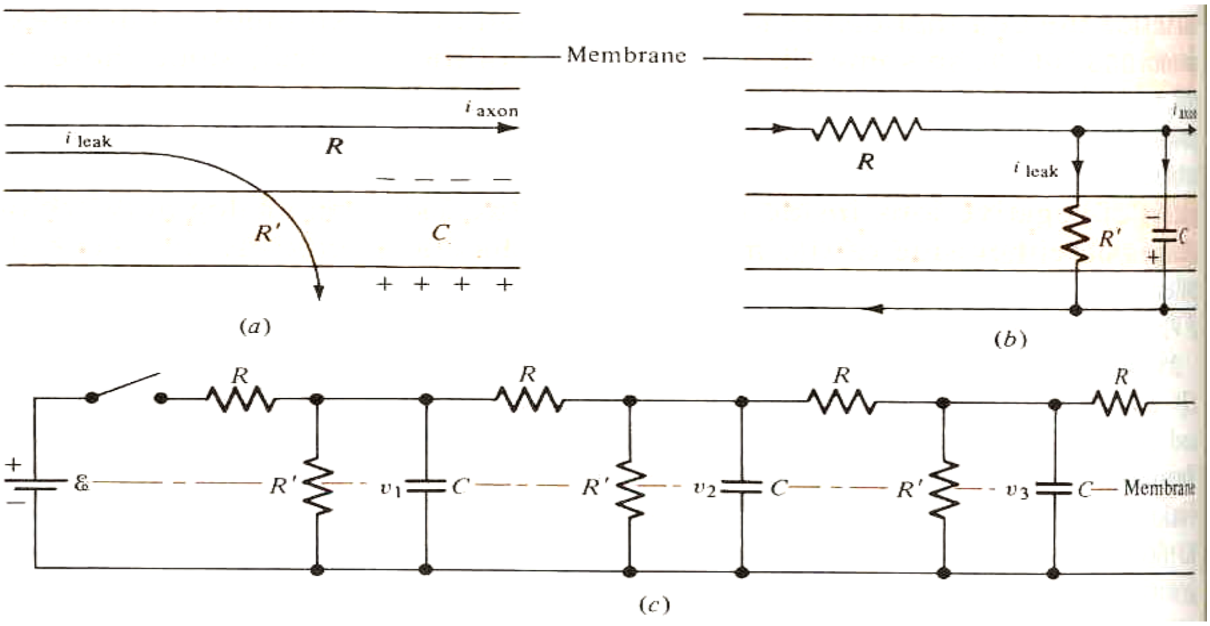

Figure 9

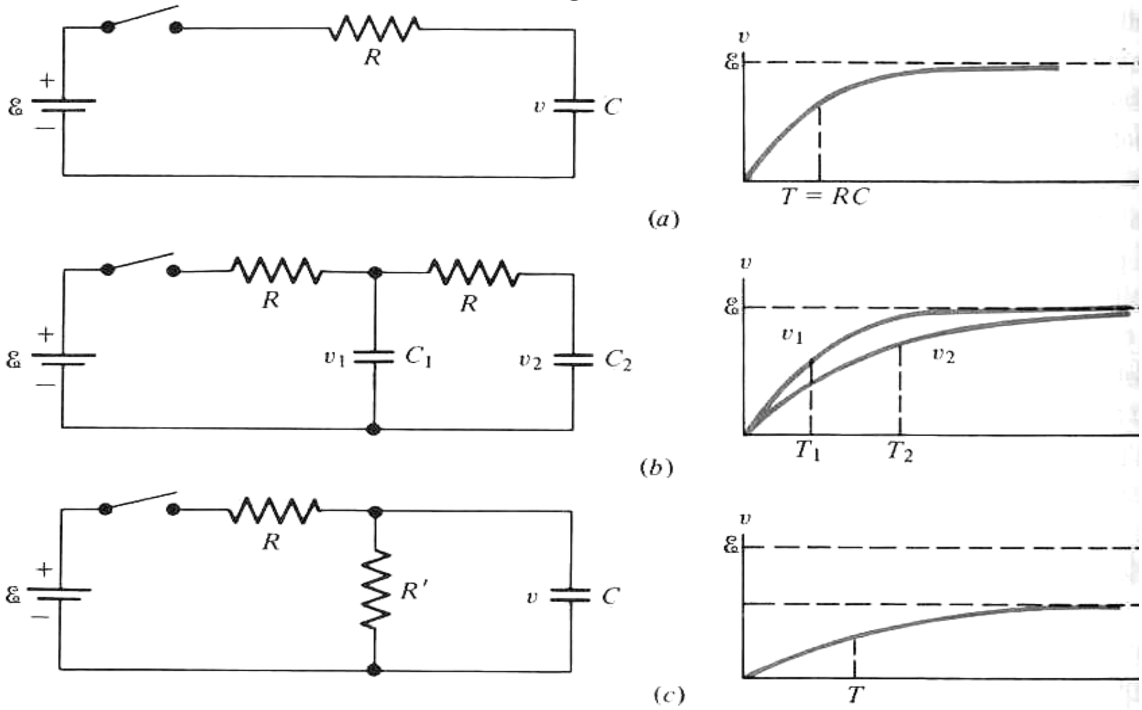

Figure 10

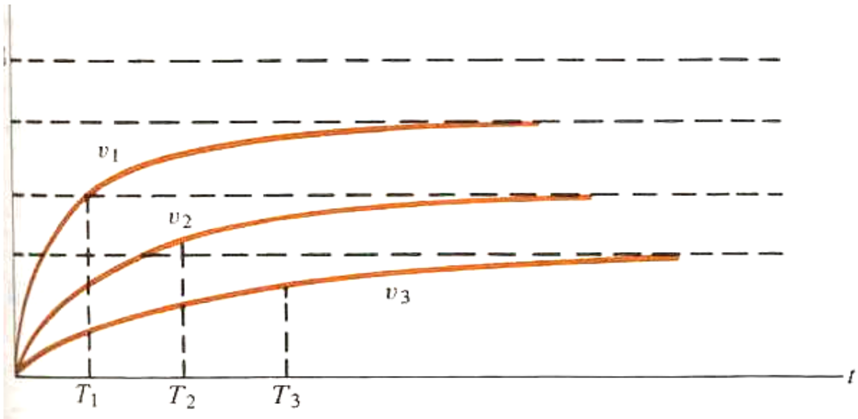

Figure 11

Very similar behavior is observed when an unmyelinated axon is stimulated weakly, as in the figure 12. A probe connected to a battery is inserted at **x=0**, and gradually the axon potential $\mathbf{V_i}$ at that point changes from –90 to –60 mV. The time required for this change to occur is determined by the membrane capacitance and the external series resistance **r**. At other values of **x**, the potentials change more slowly, reaching a final potential between –90 and –60 mV. As in the axon analog circuit, the time needed to change the potential appreciably increases with the distance **x** from the stimulus, reflecting the time needed to alter the charges on the membrane. The final magnitude of the potential changes diminishes as **x** increases because of the leakage of current through the membrane. Thus the effect of a weak stimulus propagate rather slowly and become negligible after a few millimeters.

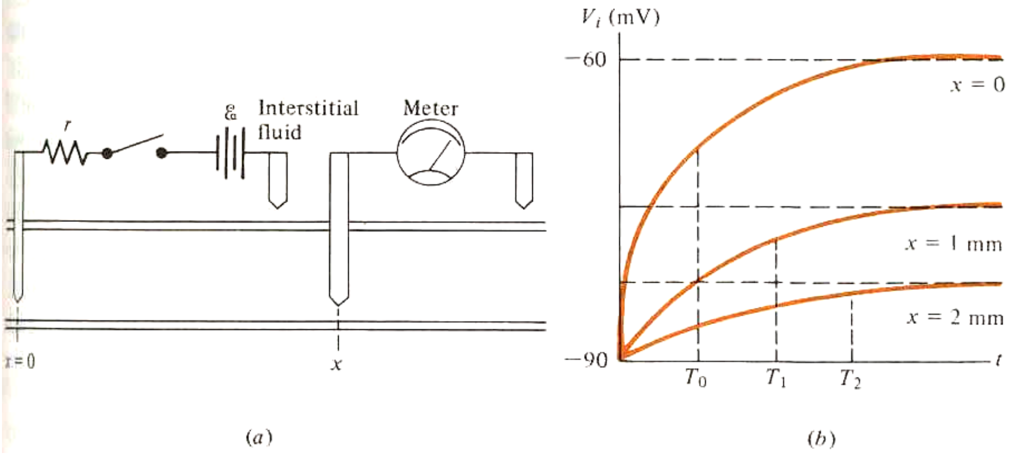

Figure 12

We have used the axon analog circuit to make quantitative predictions of the response of an axon to a weak stimulus. The analog circuit can also yield quantitative prediction relating the final axon potential at a distance x from the stimulus to the space parameter **λ.** This is done by applying **Ohm's law** to the current leaking through the membrane and traveling along axon.If the difference between the final and resting potentials at **x = 0** is $\mathbf{V_d}$, then the difference at **x** is found to be $\mathbf{V(x) = V_d e^{-x/\lambda}}$**.** This predicted dependence on **x** agrees with experimental data.