**Theme 1.** Physics principles of specification and quantification of musculoskeletal structures motor function and human body tissues mechanical properties

## Problem

Human musculoskeletal structures and tissues conditions definitive diagnostic test

**Attendance prerequisite**

**Note! Answer in writing to perform**

**1. Define or explain:** force, work, energy, and power; the principle of moments and the resolution of forces; tension stress and strain; compression stress and strain; linear limit; elastic limit; ultimate tension strength; brittle; ductile; fatigue; shear stress and strain

## Information resources

| № | Author(s) | Name of the source (textbook, manual, monograph, etc) | City, publishing house |
|---|---|---|---|
| 1 | R. M. Berne, M. N. Levy | Physiology | St Louis: Mosby Company, 1983 |
| 2 | Vander, Sherman, Luciano | Human Physiology The Mechanisms of Body Function | New York: McGraw-Hill Book Company, 1980 |
| 3 | Vander, Sherman, Luciano | Human Physiology The Mechanisms of Body Function | New York: McGraw-Hill Book Company, 1985 |
| 4 | N. V. Pronina | Biological Physics The Second Module Lectures | Simferopol, 2006 |
| 5 | Douglas C. Giancoli | Physics Principles with Applications | Pearson Education Limited; 7th Edition, 2016 |
| 6 | Martin Hollins | Medical Physics | Tomas Nelson & Sons, 1992 |
| 7 | I. Tarjan | An Introduction to Physics with Medical Orientation | Akademiai Kiado, Budapest, 1987 |
| 8 | Joseph W. Kane, Morton M. Sternheim | Physics | John Wiley & Sons Third Edition, 1988 |
| 9 | John Bullock, Joseph Boyle, Michael Wang | Physiology | Williams & Wilkins Third Edition, 1994 |

## Introduction

### Lever action of muscles and bones

**Torques.** Skeletal muscle contraction maintains posture and produces movement of the bones of the skeleton. In all cases the type of the movement of the bones is rotation. Therefore, it is possible to estimate the movement of the bones on the base of the laws of the rotation.

Suppose an object is subjected to two equal but opposite forces. The net force is zero. So, the object is in translational equilibrium. Nevertheless, it may not be in rotational equilibrium. In addition of $\mathbf{F} = 0$ we require another equilibrium condition to exclude the possibility of rotational motion.

The quantity that indicates the ability of the force to cause rotation is called **torque**.

The rigid body is in rotational equilibrium when there is no net torque acting on it.

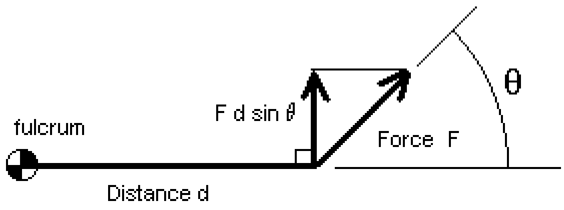



**Figure 1**

The torque **τ** depends on the force **F**, the distance **r** from a point of the axis of rotation to the point where the acts on the object, and the angle **θ** between the distance **d** and force **F**. The magnitude of the torque about fulcrum is **τ = d·F·sinθ** (fig. 1)

The dimensions of a torque are force time length so the S.I torque unit is (Newton·meter). The direction of **τ** is given by the right-hand rule and indicates the axis about which rotation will tend to occur. To illustrate the right-hand rule suppose **r** is in + x direction and **F** is in + y direction. Using the right-hand rule, we point the fingers of our right hand in the + x direction.

When our palm faces the + y direction, and our thumb is out of the pages, we can rotate our fingers $90^0$ toward the + y direction. Thus **τ** is out of the page.

**Equilibrium of the rigid bodies**

A pair of forces with equal magnitudes but opposite directions acting along different lines of action is called a **couple**. The pair of forces applied to the body do not exert a net torque.
There are **two conditions for the equilibrium of a rigid body:**

**- the net force on the object must be zero;**

**- the net torque on the object computed about any convenient point must be zero.**

These two conditions ensure that a rigid body will be both translational and rotational equilibrium.

**Levers: mechanical advantage**

**A liver in its simplest form is a rigid bar used with a fulcrum**. A liver is example of machines. In this case a force $\mathbf{F_a}$ is applied and load force $\mathbf{F_L}$ is balanced (fig. 2). The mechanical advantage (M. A.) of the machine is defined as a ratio of the magnitudes of these forces: M.A. = $F_L/F_a$. When the forces are perpendicular to a lever, its mechanical advantage is M. A. = $F_L/F_a = x_a/x_L$.

Many examples of levers are found in the bodies of animals (fig. 2). Muscles provide the forces for using these levers. Three classes of levers (I, II and III) are defined according to the relative position of $\mathbf{F_a}$, $\mathbf{F_L}$ and the fulcrum.

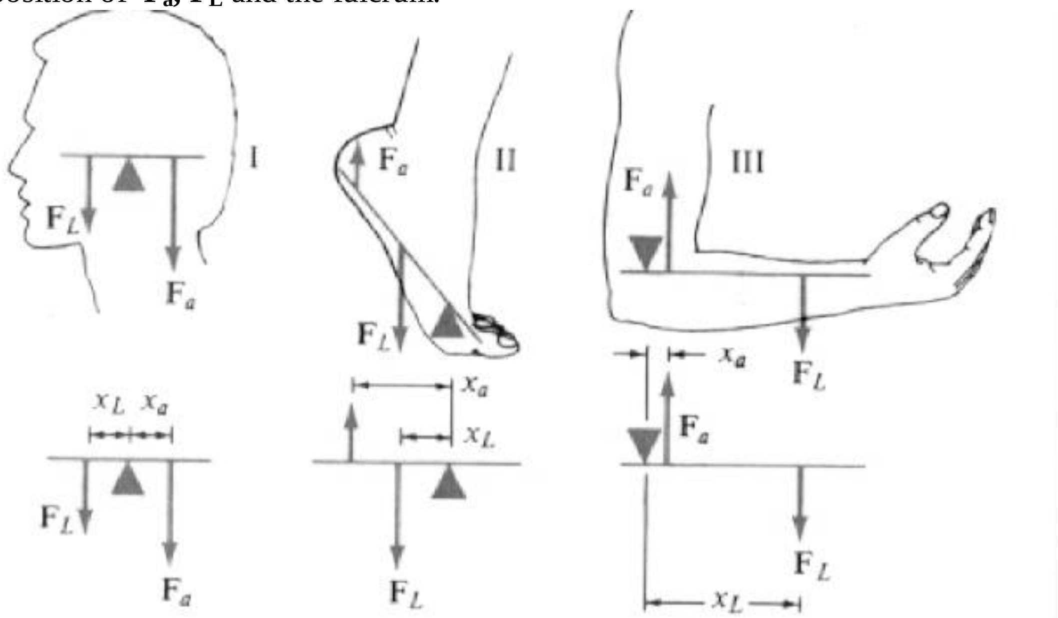



**Figure 2**

**Example**

How much force must the biceps muscle exert when a 1.0-kg mass is held in the hand with the arm outstretched as in figure 3? Assume the mass of forearm and hand together is 2.0 kg and their cg is as shown.

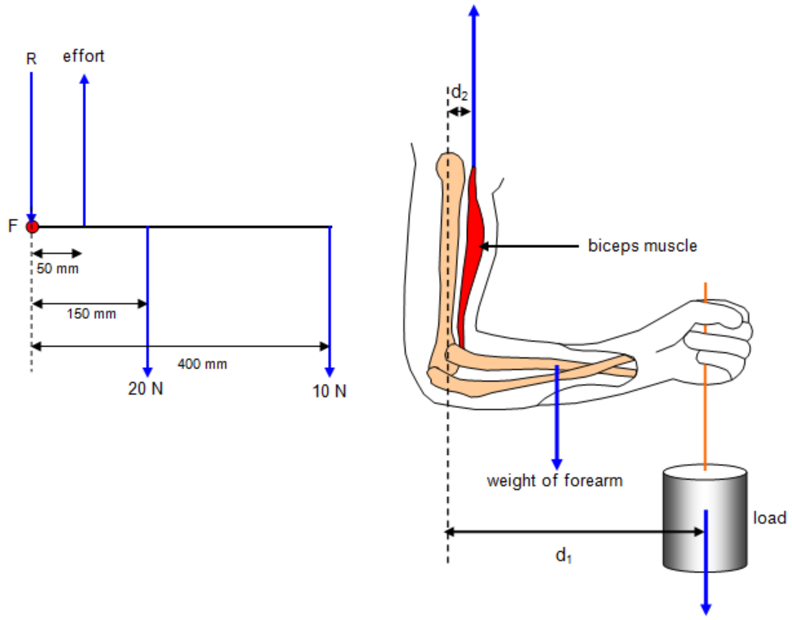



**Figure 3**

Given

$m = 1.0\ kg$

$M = 2.0\ kg$

$x_m = 40\ cm$

$x_M = 15\ cm$

$x_{F_M} = 5\ cm$

$F_M - ?$

Solution

The forces acting on the forearm are shown in figure 3 and include the upward force $F_M$ (effort) exerted by the muscle and a force R exerted at the joint by the bone in the upper arm. We wish to find $F_M$, which is done most easily by using the torque equation, calculated about the joint so that R does not enter:

$(0.05\ m)(F_M) - (0.15\ m)(2\ kg)(g) - (0.40\ m)(5\ kg)(g) = 0$.

We solve this for $F_M$ and find $F_M = 600\ N$.

**General aspects of stress and strain**

An object made from any real material will always be deformed at least slightly and may even break when forces or torques are applied.

Although materials are held together by complicated electric and magnetic forces among the molecules, the effects of these forces can be categorized quite adequately using a few measured quantities.

The deformations of materials are determined by the force per unit area, and not by the total force. Because of this it is useful to define **the stress in the bar of cross-sectional area A subjected to a force F as the ratio of the force to the area σ = F/A.** The stress is opposed by the intermolecular forces within the material.

**Three kinds of stress are commonly defined.**

**Tension stress** is the force per unit area producing elongation of an object.

**Compression stress** acts to compress an object

**Shear stress** corresponds to the application of scissor like forces.

The change in the length of the bar under tension or compression stress is proportional to its length. The strain ε is the fractional change in length **ε = Δl/l.**

**There are three kinds of strains: tension, compression, and shear.** Any deformation of an object can be considered as a combination of these three strains.

The relation between the stress and the strain for a material under tension can be found experimentally. Typical results are shown in figure 4.

The elastic deformations of a solid are related to associated stresses by quantities called elastic moduli. In the linear region of the stress-strain graph for tension or compression, the slope equals the stress-to-strain ratio and is called the **Young's modulus E** of the material: $\mathbf{E = \sigma / \varepsilon}$.

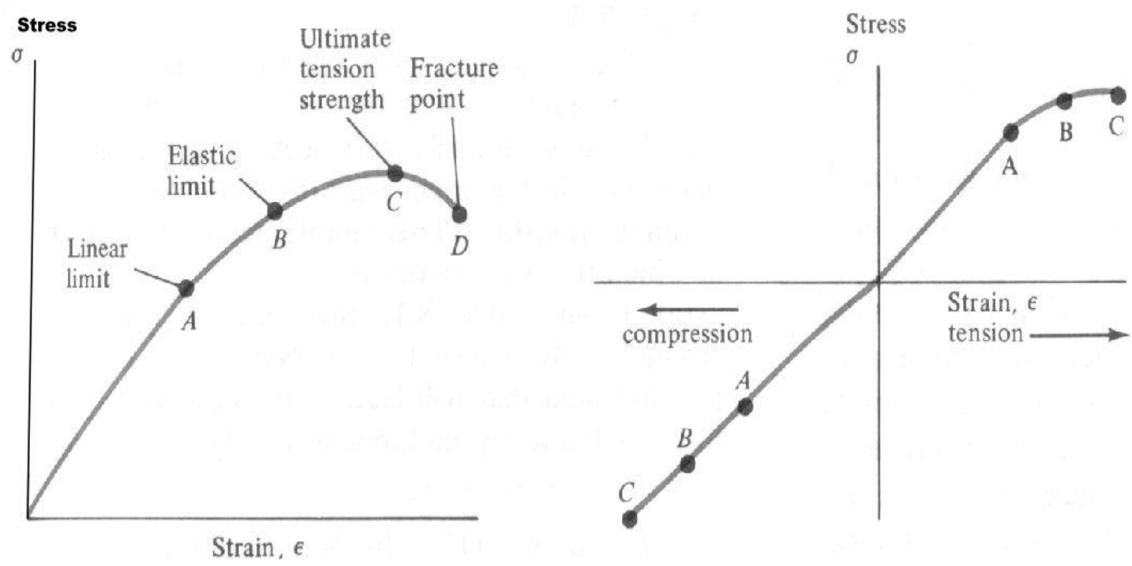



**Figure 4**

For inhomogeneous materials such as bone, the moduli for compression and tension are different. The linear stress-strain region is also called the **Hooke law region.** In this region, since the stress is linearly related to the strain, the force is linearly related to the elongation. This can be seen using the definition of Young modulus rewritten as $\mathbf{\sigma = E\ \varepsilon}$. With the definitions of the stress $\mathbf{\sigma = F/A}$ and the strain $\mathbf{\varepsilon = \Delta l/l}$, this becomes $\mathbf{F/A = E \cdot \Delta l/l}$. Thus, in tension or compression the force on an object is proportional to its elongation, $\mathbf{F = k\ \Delta l/l}$, where **k** is called the **spring constant**, and $\mathbf{k = EA/l}$. Equation $\mathbf{F = k \cdot \Delta l/l}$ is called **Hooke law**. As long as an object under stress is in the linear region, Hooke law is valid.

**Bending strength**

Figure 5 shows a bar of length **l** and rectangular cross-section with sides **a** and **b**. Placed on two supports, it bends somewhat under its own weight. When the bar bends with a radius of curvature **R**, the internal torque **τ** in the bar is given by $\mathbf{\tau = E \cdot I_A / R}$, where E is Young's modulus for the material, and $I_A$ is called the **area moment of inertia**. For rectangular bar the area moment of inertia is $I_A = a^3 \cdot b / 12$.

Many results suggest that to construct strong, light structural members, most of the material should be located as far as practical from the neutral surface.

Nature has made extensive use of the principle that hollow structures are stiffer than solid ones of the same cross-sectional area. Bones are generally hollow. For example in the human femur the ratio of inner and outer radii is about 0.5 and the cross-sectional area is only 78 percent of that of a solid bone with the same bending strength.

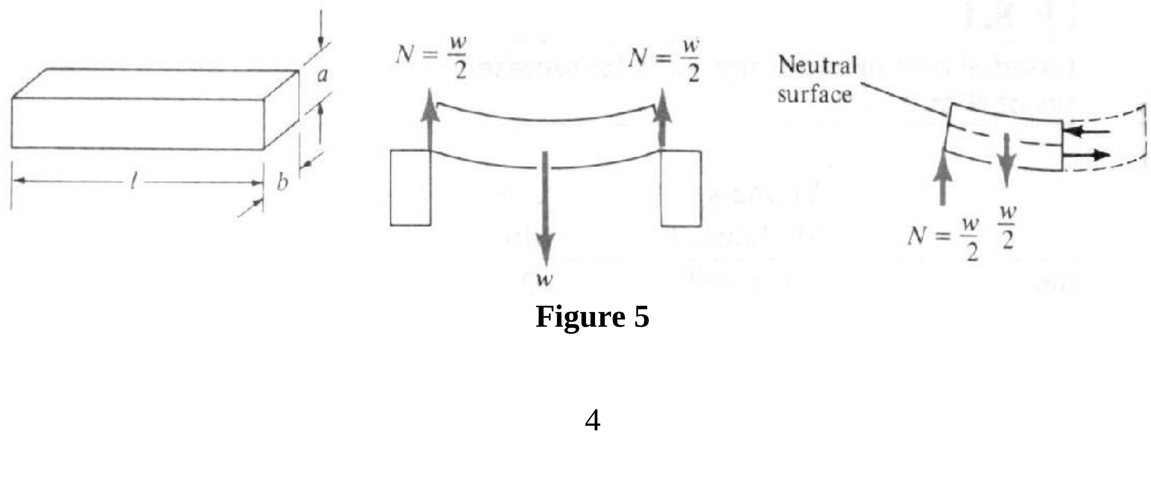



**Figure 5**

## Shearing and twisting torques

A simple example of shearing stresses and strains is provided by placing a book on a table and exerting equally large forces in opposite directions on its covers. Each page moves slightly relative to the next one, and the shape of the book changes even though its height **h** and width **w** stay nearly the same.

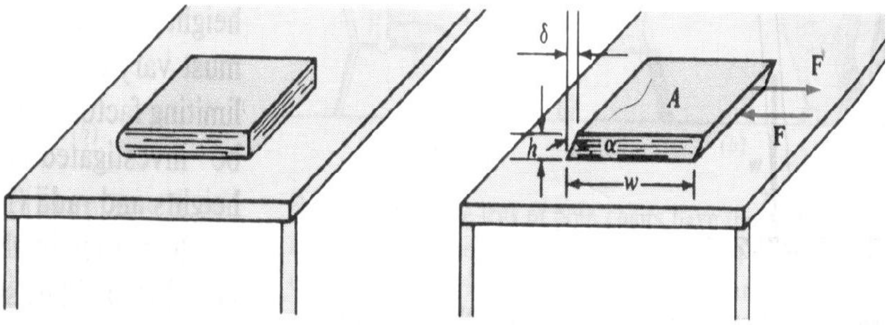
In figure 6 the book is deformed through an angle $\alpha$.The upper cover moves a distance $\delta$ relative to the lower one. The **shear stress** on the upper cover is $\boldsymbol{\delta_s = F/A}$. The **shear strain** is $\boldsymbol{\varepsilon_s = \delta/h = \tan\alpha}$. The ratio of these quantities defines the **shear modulus**, $\boldsymbol{G = \delta_s/\varepsilon_s}$.

**Twisting torques.** Figure 7 shows a cylinder fixed at one end. A couple is applied at the free end, so that there is a torque directed along the axis. If the resulting deformation is not too large, it is found that a plane drawn along the axis of the cylinder becomes twisted. The angle of twist increases linearly with the distance from the fixed end, so that the radial lines remain straight. Lines originally drawn along the outside of the cylinder parallel to the axis become slightly curved.

To find the relation between the torque $\tau$ and the deformation $\alpha$, it is necessary to compute the stress $\delta_s$ at various distances from the cylinder axis. Because adjacent cylindrical layers are subjected to shear forces, the stress $\delta_s$ and strain $\varepsilon_s$ are related by $\boldsymbol{\delta_s = G \cdot \varepsilon_s}$. The torques on each layer are computed and summed, it is found that if $\alpha$ is expressed in radians,

$$\tau = G \cdot I_p \cdot \alpha / l.$$

This is similar in form to the result for bending torques, but $\boldsymbol{I_p}$ is **the polar moment of inertia.** For a cylinder of radius r, $I_p = \pi \cdot r^4/2$ (solid cylinder).




**Fugure 6**

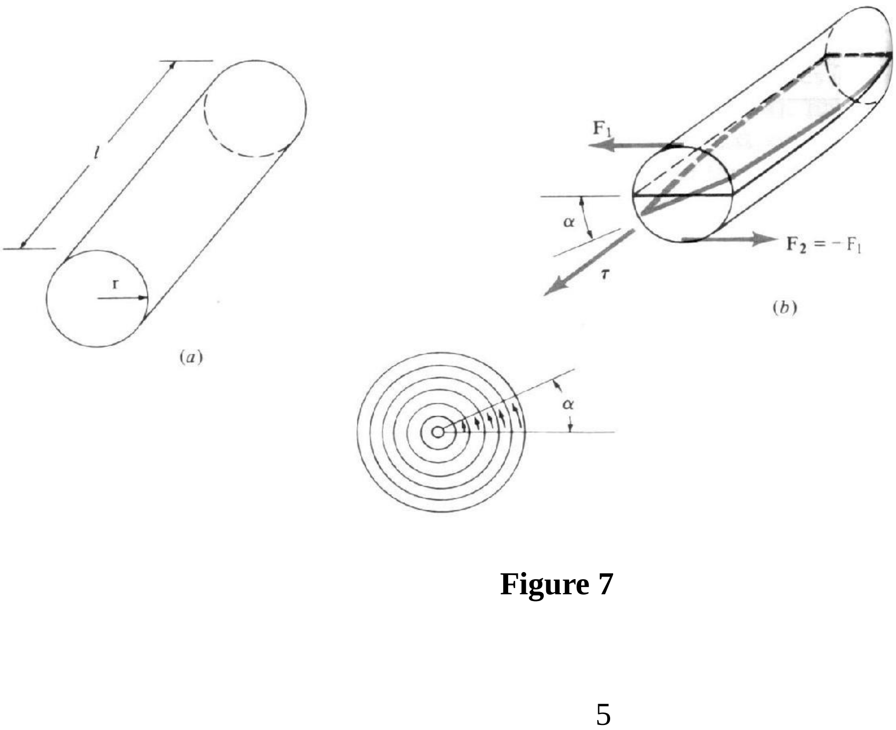



**Figure 7**

**Example**

Two bones of equal radius are subjected to equal angles of twisting. If one is longer than the other, which will fracture first?

Given

$r_1 = r_2$

$\alpha_1 = \alpha_2$

$l_2 > l_1$

Solution

The twisting torques on both are $\tau_1 = \frac{G_1 I_{p_1} \alpha_1}{l_1}$ and $\tau_2 = \frac{G_2 I_{p_2} \alpha_2}{l_2}$.

The shear moduli are the same $G_1 = G_2$ because the bones are made of the same material and subjected to equal deformations; the moments of inertia are the same $I_{p_1} = I_{p_2}$ as it depends on the radius only $I_p = \frac{\pi r^4}{2}$ and the radii are the same $r_1 = r_2$; the angles are the same $\alpha_1 = \alpha_2$, so $\tau_1 l_1 = \tau_2 l_2$, $\frac{\tau_1}{\tau_2} = \frac{l_2}{l_1} > 1$, then $\tau_1 > \tau_2$, respectively, the first (shorter) body fractures first.

**Review questions**
**Note! Answer in writing to perform**

**1**. If a force F is applied to a bar of cross section A, the stress is__________.
**2**. The strain in an object subjected to a stress is the______________.
**3**. The three types of stress are________, __________, and___________.
**4**. In the linear region, the___________and___________are linearly proportional.
**5**. Up to the __________, an object returns to its original length when the stress is removed.
**6**. If a material is easy to compress, it has a small___________.
**7**. If the force needed to stretch an object is proportional to the elongation, the object obeys_____________.
**8**. A bar with a large area moment of inertia is__________ to bend than one with a small area moment of inertia.
**9**. Buckling of a column refers to collapse under forces approximately along its____________.
**10**. The shear modulus is the ratio of the_________to_____________.
**11**. Twisting a cylinder produces____________stresses.